



LITTORAL MATHEMATICS TEACHERS ASSOCIATION (L.M.T.A.)



GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION

0770 PURE MATHS WITH STATISTICS 2

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ADVANCED LEVEL

Subject Title	Pure Mathematics With Statistics
Paper No.	2
Subject Code No.	0770

THREE HOURS

Full marks may be obtained for answers to ALL questions.

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1.

- (a) Show that one root of the equation $3x^3 - x = 1$ lies in the interval $0 < x < 1$. Taking 0.75 as the first approximation to the root of this equation, use one iteration of the Newton Raphson's procedure to find to 3 decimal places a better approximation to the root. **5 marks**
- (b) Find the number of arrangements of all the letters of the word SYLLABUSES such that each arrangement must contain the word BUS. **2 marks**

2. Given that $f(x) = 4\sin 3x - 3\cos 3x$, express $f(x)$ in the form $R\cos(3x + \lambda)$ where $R > 0$ and λ is an acute angle. Hence find

- (i) the maximum value of $\frac{5}{|f(x)| + 5}$. **5 marks**
- (ii) all solutions to the equation $f(x) - 2.5 = 0$ in the interval $0 \leq x \leq 360^\circ$. **3 marks**

3.

- (a) p and q are two statements represented by:
 p : Paul is doing Philosophy
 q : Paul is abnormal
 Represent in ordinary English the statement represented by $\sim(p \Rightarrow q)$. **1 mark**
- (b) Prove by
- (i) contradiction that the statement “ $(2, 3)$ is the center of the circle $(x - 2)^2 + (y - 3)^2 = 1$ ” is true. **4 marks**
- (ii) Mathematical induction that

$$\sum_{r=1}^n (2r-1)^2 = \frac{4n^3 - n}{3}, \forall n \in \mathbb{N}. \quad \textbf{4 marks}$$

4.

- (a) Given that z is a complex number such that $\frac{z-i}{1+z} = -i-1$. Find
- (i) z in the form $a+bi$ where a and b are constants. **3 marks**
- (ii) $|z^2|$ and $\arg(z^2)$. **3 marks**
- (b) Solve for z in the form $a+bi$ where a and b are constants in the equation $z^3 - 1 + i = 0$. **4 marks**

5.

Given the matrix

$$\mathbf{M} = \begin{pmatrix} -1 & 3 & 5 \\ 0 & 1 & 2 \\ 3 & 1 & k \end{pmatrix}.$$

- (i) Find the value of k for which the matrix $\mathbf{M}$ is a singular matrix. **3 marks**
- (ii) For $k = 4$, find $\mathbf{M}^{-1}$, the inverse of $\mathbf{M}$ and the point whose image is $(2, -1, 3)$ under the transformation matrix $\mathbf{M}$. **7 marks**

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(a) Find $\frac{dy}{dx}$ given that

(i) $y = \ln\left(\frac{\sin x}{1 + \cos x}\right)$ 2 marks

(ii) $y = \sqrt[3]{(5x^2 + 1)}$ 2 marks

(b) Given the functions $f(x) = \frac{x}{x^2 - 1}$

(i) Find the monotonicity of f , showing clearly its variation table. 2 marks

(ii) Sketch the graph of $y = f(x)$, showing clearly all its asymptotes and the behavior of the curve as it approaches its asymptotes. 5 marks

7. The vector equation of two lines l_1 and l_2 are

$$l_1 : \frac{1+x}{2} = \frac{2-y}{3} = \frac{-3+z}{4} \text{ and}$$

$$l_2 : \frac{x-1}{2} = a-y = \frac{z+3}{2}.$$

Given that l_1 and l_2 intersect at the point P , find

(i) the value of a . 4 marks

(ii) the coordinates of the point P . 1 mark

(iii) the parametric equation of the plane π_1 containing lines l_1 and l_2 , in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b} + \mu\mathbf{c}$. 2 marks

(iv) the Cartesian equation of the plane π_2 containing the point P , $A(2, 3, -1)$ and $B(1, -1, 1)$. 4 marks

8.

(a) Resolve $\frac{2x-3}{x^3+x}$ into partial fractions. Hence evaluate $\int \frac{2x-3}{x^3+x} dx$. 5 marks

(b) Find the range of values of x for which $x-3 < \frac{2(x+3)}{x}$ 3 marks

(c) Given that the roots of the quadratic equation $x^2 + px - q = 0$ differ by 3, show that $p^2 + 4q - q = 0$. 4 marks

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(a) A polynomial $f(x) = ax^3 - bx^2 - 2x + 3$ is exactly divisible by $2x^2 - x - 3$. Find the values of a and b , hence find the remainder when $f(x)$ divides $(x+2)$ 5 marks

(b) Given that $(1 + \cos 2x)e^y \frac{dy}{dx} - 2(1 + e^y) \sin 2x = 0$ and that $y = 0$ when $x = \frac{\pi}{4}$.
Show that $y = 2 \ln(\tan x)$. 5 marks

(c) Use the trapezium rule with 3 ordinates to find an approximate value of $\int_1^{49} \frac{1}{3 + \sqrt{x}} dx$. 3 marks

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10. Visit www.alstug.com for more papers, acedic blogs and notes

(a) Given the function $f : x \rightarrow \frac{a}{b-x}$, if $f(2) = 1$ and $f^{-1}(-1) = 3$, find the values of a and b .

3 marks

(b)

(i) Show that the first four terms of the binomial expansion of $\sqrt{\frac{1-2x}{1+3x}}$ are

$$1 - \frac{5}{2}x + \frac{35}{8}x^2 - \frac{185}{16}x^3 \dots$$

4 marks

(ii) Find the range of values of x for which the expansion is valid.

2 marks

(c) A binary relation R is defined in $\mathbb{Z}$, the set of integer such that

$$\forall a, b \in \mathbb{Z}, aRb \Rightarrow a = b^{\frac{1}{n}}, n \in \mathbb{N}^*.$$

Show that R is a partial ordered relation.

5 marks

END!

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