

Pure Maths With Stats 3

0770/3

GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2024

ADVANCED LEVEL

Subject Title	Pure Mathematics With Statistics
Subject Code No.	0770
Paper No.	Paper 3

Duration: Three Hours

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and tables produced by the GCE Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Electronic calculators may be used.

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1. (i) Two independent events A and B are such that $P(A) = \frac{1}{5}$ and $P(B) = \frac{3}{20}$.

Find

- (a) $P(A/B)$,
- (b) $P(A \cap B)$,
- (c) $P(A \cup B)$.

(7 marks)

- (ii) 60% of the population of a certain country have been vaccinated against COVID-19.

During an outbreak of the COVID-19 epidemic, $\frac{1}{5}$ of those who had taken the vaccine were infected and $\frac{2}{3}$ of those who had not taken the vaccine were infected.

Find the probability that an individual selected at random from this country

- (d) is infected.
- (e) is infected or had taken the vaccine.

(6 marks)

2. The lengths, to the nearest cm, of some randomly selected 140 leaves from a certain fruit tree were measured and recorded as follows:

Length in cm	5 - 9	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49
Number of leaves	11	13	17	20	25	22	16	12	4

- (a) State the modal class of this distribution of lengths.
- (b) Calculate the mean and variance of the lengths of the leaves, to 3 decimal places.
- (c) Construct the cumulative frequency table and draw the cumulative frequency curve for these data.
- (d) Use your curve to estimate the interquartile range.

(1, 6, 4, 2 marks)

3. A random variable X takes its values in the set $\{0, 1, 2, 3, 4\}$ and has the distribution

x	0	1	2	3	4
$P(X = x)$	a	a	a	b	b

Given that $P(X \geq 2) = 3P(X < 2)$,

calculate

- (a) the values of the probabilities a and b ,
- (b) the expected value of X ,
- (c) the standard deviation of X ,
- (d) the expected value of Y where $Y = 5X + 3$.

(5, 3, 3, 2 marks)



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4. The probability density function f of a continuous random variable X is given by

$$f(x) = \begin{cases} kx, & 0 < x < 1 \\ k + 1 - x, & 1 \leq x \leq 2 \\ 0 & \text{elsewhere.} \end{cases}$$

Find

- (a) the value of k .
- (b) $P\left(X > \frac{3}{2}\right)$,
- (c) $E(X)$,

Given that $Y = \alpha X + \frac{1}{3}$,

- (d) Find the value of α for which $E(Y) = \frac{7}{6}$.

(3, 3, 4, 3 marks)

5. (i) In an experiment the length l of a metal bar is measured many times and the observed values recorded.

The recorded values are approximately normally distributed with mean 1.340m and standard deviation 0.021m.

Find the probability that a recorded value

- (a) exceeds 1.37 m,
- (b) lies between 1.31m and 1.37m.

(3, 3 marks)

Given that 10% of the recorded values are less than l ,

- (c) find, to three decimal places, the value of l .

(4 marks)

- (ii) In a train station, the time T in hours between the arrival of two trains is a random variable with an exponential distribution having the density function

$$f(t) = 3e^{-3t}, \quad t > 0.$$

A train arrived at 9:00 am. What is the probability that the next train will arrive in less than 3 hours. (3 marks)

6. A company manufactures memory cards and one out of 100 cards manufactured is known to be defective.

The memory cards are packed in packs of 10.

What is the probability that

- (a) there will be at least a defective card in a randomly chosen pack?
- (b) exactly two defective cards are found in a pack?

(6, 3 marks)

The company offers a replacement guarantee if a pack contains at least two defective cards.

- (c) What percentage of packs will the company have to replace?

(4 marks)

7. (i) A large population consists of equal numbers of the digits 2 and 4.

- (a) Find the mean μ and the variance σ^2 of this population.
- (b) Construct the probability distribution of the mean, $\bar{x}$, of all possible samples of size 3 taken from this population.

(4, 3 marks)

- (ii) A machine is set to produce sweets whose masses are normally distributed with mean 15.21g and standard deviation 0.5g. To check the setting of the machine a sample of 50 sweets is taken from the production line of the machine and the mean mass is found to be 15g. Test at 3% level of significance whether the machine is correctly set.

(6 marks)



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8. The following table gives the marks scored by eight candidates in each of two Biology examinations.

Candidate	A	B	C	D	E	F	G	H
Mark in Paper 2, x	84	42	33	52	69	81	50	35
Mark in Paper 3, y	83	60	28	63	59	92	73	40

Calculate

- (a) the covariance of the data,
- (b) the equation of the least squares regression line for the mark scored in Paper 3 on the mark scored in Paper 2,
- (c) the mark in Paper 3, to the nearest whole number, of a candidate who scored 80 in Paper 2,
- (d) the Kendall's coefficient of rank correlation.

(3, 5, 2, 3 marks)

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