

0775/3/2023
F. MATHS A/L

**SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION**

THE TEACHERS' RESOURCE UNIT (TRU)

IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR SCIENCE EDUCATION

AND

**THE SOUTH-WEST ASSOCIATION OF MATHEMATICS TEACHERS
(SWAMT)**

SATURDAY AFTERNOON, 23/03/2023

ADVANCED LEVEL

Subject Title	Further Mathematics
Paper Number	Paper 3
Subject Code Number	0775

Two And A Half Hours

INSTRUCTIONS TO CANDIDATES:

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

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You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the CGCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.



1. Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are such that:

$\vec{F}_1 = (-i + 3j + k)N$ acts through the point $(3i - 3j - 3k)m$.

$|\vec{F}_2| = 6N$ and $\vec{F}_2$ acts through the point with position vector $(3i + 2j - 4k)m$, in the direction of the vector $(i + 2j - 2k)$.

$|\vec{F}_3| = 7N$ and $\vec{F}_3$ acts along the line $r = 8i + 3j + \alpha(6i + 3j + 2k)$.

- (a) Find in vector form the forces $\vec{F}_2$ and $\vec{F}_3$ (3 marks)

If $\vec{F}_3$ acts on a particle, moving it from the point where it acts to the point where $\alpha = 2$,

- (b) find the work done on the particle by $\vec{F}_3$ (3 marks)

This system of three forces is equivalent to a force $\vec{F}_4$ acting through the point with position vector $(3i - 2j - 2k)m$, together with a couple G .

- (c) Find $\vec{F}_4$ and G . (5 marks)

- (d) Find also, the magnitude of the moment of $\vec{F}_4$, about the point with position vector $(2i - 4j - 2k)$ (3 marks)

2. A new and wonderful computer has come on the market. The rate of increase in the number of people, x , who own one, is equal to the product of x at time t (in months), who have already acquired the computer and the purchasing power, P , of the population.

The purchasing power, P , is measured by the excess of the maximum number of people, k , that can acquire the computer over the number of people, x , at time t , who have already owned the computer. By treating x as a continuous variable,

- (a) obtain a differential equation relating the increase in x . (2 marks)

Hence,

- (b) show that if $x = x_0$ when $t = 0$, then at any time t , thereafter, $x = \frac{kx_0 e^{kt}}{k - x_0(1 - e^{kt})}$. (4 marks)

Given that k, P and x_0 are measured in thousands, $k = 10$ and $x_0 = 2$

- (c) find, to 2 d.p., the time in days, when $P = 4$. (Assume that there are 30 days in any month) (5 marks)

- 3(a) The finite region bounded by the curve $y = e^{\sin x}$, the x -axis and the ordinates $x = 0$ and $x = \pi$ is rotated completely about the x -axis. Use Simpson's rule with 5 ordinate to estimate, in terms of π , correct to 2 d.p., the volume of the solid generated. (4 marks)

- (b) Find, in ascending powers of x , up to and including the term in x^4 , the solution of the differential equation $\frac{d^2 y}{dx^2} + y \frac{dy}{dx} + x = 0$, given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$.

Using your solution, find, to 4 d.p., value of y when $x = 0.1$. (4 marks)

- (c) Using the approximation $h^2 \left(\frac{d^2 y}{dx^2} \right)_n \approx y_{n+1} - 2y_n + y_{n-1}$, with a step length $h = 0.1$, find the value of y , correct to 3 d.p., when $x = 1.3$, given that $\frac{d^2 y}{dx^2} = xe^y$ and that $y = 1.2$ when $x = 1$ and $y = 1.5$ when $x = 1.1$. (4 marks)

- 4.(a) The random variable X is such that $X \sim B(n, p)$.

- (i) State, in terms of n, p and q , $E(X)$ and $Var(X)$ (2 marks)

Hence,

- (ii) show that $E(X^2) = np(q + np)$ (1 mark)

- (iii) Given that $\frac{Var(X)}{E(X)} = 0.8$, and $E(X^2) = 5.6$, find the value of n (2 marks)



(iv) Hence, evaluate $P(X \geq 4)$, correct to 4 d.p.

(2 marks)

(b) The lifetime, in months, of an electronic device is a continuous random variable T . The probability that such a device lasts longer than a time t months is $e^{-t/k}$, where k is a positive real constant.

(i) Find the probability density function of such a device.

(2 marks)

Of 2500 such devices, 648 last within the first 3 months of use.

(ii) Calculate the value of k , to the nearest integer.

(3 marks)

Hence,

(iii) find to 4 d.p, the probability that one of such devices lasts longer than 5 months of use.

(2 marks)

5. Two smooth spheres A and B of different masses, moving on a smooth horizontal plane collide.

Before impact, the velocities of A and B are $(6\sqrt{2}i + 6j) \text{ ms}^{-1}$ and $3j \text{ ms}^{-1}$ respectively.

Immediately after impact, A moves in a direction perpendicular to the line of centres, and B moves in a direction making an angle of 45° with the line of centres.

(a) Find the speeds of A and B after impact

(4 marks)

(b) Show that the coefficient of restitution between A and B is $\frac{1}{2}$

(2 marks)

Given that the mass of B is 2 kg, find

(c) the mass of A,

(2 marks)

(d) the magnitude of the impulse experienced by A,

(2 marks)

(e) the loss in kinetic energy during the impact.

(2 marks)

6. A particle P moves on a polar curve such that at time t , its polar coordinates (r, θ) satisfy the equations $\dot{r} = 24\sin\theta$ and $\dot{\theta} = (3\cos\theta + 5)^2$, $0 < \theta < \pi$.

(a) Given that $r = 1$ when $\theta = 0$, show that the polar equation of the curve is $r = \frac{8}{3\cos\theta + 5}$

(4 marks)

(b) Hence, show that the value of r at the point on the curve where the particle is moving parallel to the line $\theta = 0$ is $\frac{5}{2}$

(5 marks)

(c) Find the velocity of P at this instant, and show that this velocity makes an angle α with OP, where $\tan\alpha = \frac{4}{3}$

(3 marks)

(d) Show that the radial acceleration at this time is $-\frac{2048}{5}$.

(3 marks)

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7. A particle of mass 1 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 5 N. The other end of the string is attached to a fixed point O, on the surface of a smooth horizontal table. The particle is held at a point on the table 2m from O and is released. Its motion is resisted by a force of magnitude $2v$ N, where v is the speed of the particle.

(a) Show that so long as the string remains taut, the motion of the particle is governed by the equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$, where x is the displacement of the particle from the equilibrium position at time t .

(2 marks)



Hence,

- (b) show that this displacement x at time t is given by $x = \frac{\sqrt{g}}{2} e^{-t} \cos(2t - 0.4636)$ (4 marks)
 (c) Find to 3 d.p, the time that elapses before the string becomes slack. (2 marks)
 (d) Find to 3 d.p, the speed of the particle at the instant when the string goes slack. (2 marks)
 (e) Show that the distance from O at which the particle comes to rest is approximately $0.596m$ (3 marks).

8. A plane composite body in the form of a hand fan is formed by attaching a thin uniform rod of mass $3m$ and length $2a$ to a uniform circular disc of mass $2m$ and radius a . The composite body is free to rotate in its own plane, about a fixed smooth horizontal axis perpendicular to its plane, and passing through the midpoint of the rod.

- (a) Show that the moment of inertia of the composite body about this axis is $10ma^2$ (4 marks)
 (b) Find the radius of gyration of the body about this axis. (2 marks)

While in its position of stable equilibrium, the body is given a small horizontal push on the disc, so that it oscillates through small angles in a vertical plane.

- (c) Show that its equation of motion can be expressed in the form $5a \left(\frac{d\theta}{dt} \right)^2 = 4g(\cos\theta - 1)$, (5 marks)
 where θ is the angle through which the body turns in time t seconds.
 (d) Find the period of small oscillations and the length of the equivalent simple pendulum. (4 marks)

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