

0775/2/2024
F. MATHS A/L

**SOUTH WEST REGIONAL MOCK EXAMINATION
GENERAL EDUCATION**

THE TEACHERS' RESOURCE UNIT (TRU)

IN COLLABORATION WITH

THE REGIONAL INSPECTORATE OF PEDAGOGY FOR SCIENCE EDUCATION

AND

THE SOUTH-WEST ASSOCIATION OF MATHEMATICS TEACHERS

(SWAMT)

FRIDAY AFTERNOON, 22/03/2024

ADVANCED LEVEL

Subject Title	Further Mathematics
Paper Number	Paper 2
Subject Code Number	0775

THREE HOURS

INSTRUCTIONS TO CANDIDATES:

Answer ALL questions.

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For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae and tables published by the CGCE Board and noiseless, non-programmable calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.



- 1 (a) Find the complementary function of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3e^{-x}$, in the form $y = f(x)$. (3 marks)

Hence,

- (b) find the particular integral and the general solution in the form $y = f(x)$. (4 marks)

- 2 (a) Express in partial fractions the function $f(x) = \frac{2x^3 + x + 2}{(x^2 + 1)(x^2 + 2)}$ (4 marks)

Hence or otherwise,

- (b) Show that $\int_0^1 f(x) = \frac{\pi}{2} - 2\ln 2 + \frac{3}{2}\ln 3 - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$ (5 marks)

- 3 (a) Solve for x , the equation $\tanh^{-1}\left(\frac{x+1}{x-1}\right) = \ln 3$ (4 marks)

- (b) Show that the set $S = \{1, 3, 5, 7\}$ forms a group under multiplication modulo 8 ($\times_8$) (4 marks)

- 4 (a) A linear transformation is represented by the matrix $M = \begin{pmatrix} 1 & -1 & 1 \\ 3 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$.

- (i) Find the image of the line $r = 2i - 3j - 4k + t(i + j - k)$, under M . (3 marks)

- (ii) Find the image of the plane $x + y - z = 3$, under M . (3 marks)

- (b) A curve is given by the parametric equations $x = \theta - \sin\theta$, $y = \cos\theta$, $0 \leq \theta \leq \frac{\pi}{3}$.

Show that the length of the curve is $4 - 2\sqrt{3}$. (4 marks)

- 5 (a) Show that the curve with polar coordinates (r, θ) , where $r = \frac{36}{9 - 5\cos^2\theta}$, is an ellipse, E , in the (x, y) plane. Find the eccentricity of E . (5 marks)

- (b) Show that the point $P(3\cos\theta, 2\sin\theta)$ lies on E . Also find the equation of the normal to E at the point P . (4 marks)

Given that the normal at P to E meets the coordinate axes at the points A and B ,

- (c) show that when $\theta = \frac{\pi}{4}$, the area of triangle AOB is $\frac{25}{24}$ squared units. (4 marks)

- 6 (a) Solve the system of congruences $x \equiv 3 \pmod{5}$, $x \equiv 4 \pmod{7}$ (4 marks)

- (b) Using De Moivre's theorem, or otherwise, show that

$$\sum_{r=0}^{\infty} \left(\frac{1}{4^r}\right) \cos 2r\theta = \frac{4(4 - \cos 2\theta)}{17 - 8\cos 2\theta} \quad \text{Meet us at } \text{www.alstug.com} \quad (4 \text{ marks})$$

7. A transformation, f , on a complex plane is defined by $z' = 3z + 2 - 4i$.

- (a) Find the image of the point $z = 3 - i$ (1 mark)

- (b) Determine the invariant point of f in the form $a + bi$, $a, b \in \mathbb{R}$ (2 marks)

- (c) Show that f is a similarity transformation and state its radius. (3 marks)

- (d) Give the geometric interpretation of f . (2 marks)

8. Consider the two vectors $\vec{a} = (3 - \alpha)i - j - 5k$ and $\vec{b} = (2 + \alpha)i + 3j + (2 + \mu)k$, $\alpha, \mu \in \mathbb{Z}$.

Given that $\vec{a} \times \vec{b} = 11i - 2j + 5k$.

- (a) Find the values of the real constants α and μ . (3 marks)

Hence,

- (b) State the vectors $\vec{a}$ and $\vec{b}$. (2 marks)

- (c) Show that $\vec{a}$ and $\vec{b}$ are linearly independent. (2 marks)



- (d) Find the cartesian equation of the plane which contains $\vec{a}$ and $\vec{b}$ and which passes through the point $P(1, -3, 7)$. (2 marks)

9. A function f , is defined by $f(x) = \frac{x^3}{(x-1)^2}$
- (a) Find the domain of f . (1 mark)
 - (b) Find the axes intercepts of the curve $y = f(x)$. (1 mark)
 - (c) Find the equations of the two asymptotes to the curve $y = f(x)$. (3 marks)
 - (d) Determine the stationary points on the curve $y = f(x)$. (3 marks)
 - (e) Obtain a variation table for f and state its monotonicity. (5 marks)
 - (f) Sketch the graph of $y = f(x)$. (3 marks)

10. The sequences (U_n) , (V_n) and (W_n) are defined by
 $U_0 = 2$, $U_{n+1} = \frac{1}{4}(5V_n - U_n)$; $V_0 = 11$, $V_{n+1} = \frac{1}{3}(4V_n - U_n)$; $W_n = V_n - U_n$
- (a) Find U_1 and V_1 . (3 marks)
 - (b) Show that (W_n) is a convergent sequence with positive terms. (5 marks)
 - (c) Find (W_n) in terms of n and state its limits. (3 marks)
 - (d) Show that (U_n) and (V_n) are both increasing. (4 marks)

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