



LITTORAL MATHEMATICS TEACHERS ASSOCIATION (L.I.M.T.A)



GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION

0775 FURTHER MATHEMATICS 2

APRIL 2022

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ADVANCED LEVEL

Subject Title	Further Mathematics
Paper No.	2
Subject Code No.	0775

THREE HOURS

Full marks may be obtained for answers to ALL questions.

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. The differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^{-3x},$$

has general solution $y = pe^{2x} + qe^{-3x} + f(x)$, where $f(x)$ is a particular integral satisfying the above differential equation

(i) Find $f(x)$ 4marks

(ii) Given that $y = -1, \frac{dy}{dx} = 1$ when $x = 0$, find the solution of the differential equation 3marks

2.

(i) A cyclic group $G(\{1, -1, i, -i\}, \times_{\mathbb{C}})$, where $\times_{\mathbb{C}}$ denotes multiplication over the set $\mathbb{C}$.

(a) Show that the element i is a generator element of G , stating its order. 3marks

(b) Another group $S = (\{1, 3, 7, 9\}, \times_{10})$ where $\times_{10}$ denotes multiplication modulo 10. Show that G and S are isomorphic. 3marks

(ii) Using the Chinese remainder theorem or otherwise, solve the system of congruences

$$2x \equiv 4 \pmod{5}$$

$$x + 3 \equiv 1 \pmod{3}$$

4marks

3. Given that $f(x) = \frac{x+2}{x^2(x^2+2)}$

(i) Express $f(x)$ in partial fractions 3marks

(ii) Hence prove that $\int_1^{\sqrt{2}} f(x) dx = \frac{1}{4} \ln\left(\frac{3}{2}\right) + \frac{\sqrt{2}-1}{\sqrt{2}} - \frac{\pi}{4} + \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ 4marks

4.

(i) A curve is defined by $y^2 = \frac{x^3}{a}$, where a is a constant, for $0 \leq x \leq \frac{7a}{3}$. Show that the arc length of the curve in units is $\frac{13a}{3}$. 5marks

(ii) Find the value of the constant k for which the vectors a, b and c given by $a = i + 2j + 3k, b = 2i + j + 2k$ and $c = -2i + kj + 6k$ are linearly dependent. 3marks

5.

(i) Consider the curve C with polar equation $r^2 = 2 \operatorname{cosec} 2\theta$, Meet us at www.alstug.com

(a) Show that the Cartesian equation of this curve is a rectangular hyperbola 2marks

(b) Show that the equation of tangent at the point $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$ is $3y + x = 2\sqrt{3}$ 3marks

(ii) An ellipse has its foci on the y -axis and its centre at the origin. The distance between the foci is 6 and the major axis is of length 10. Find the eccentricity and the Cartesian equation of this ellipse. 4marks

6. Express $7 \cosh x - 5 \sinh x$ in the form $R \cosh(x - \ln p)$, where R and p are positive constants to be determined.

Hence solve the equation $7 \cosh x - 5 \sinh x - 3\sqrt{6} = 0$ 6marks

7.

(i) Given that $I_n = \int_0^1 x^n \sqrt{1-x^2} dx; \forall n \geq 0$,

(a) Show that $I_{n+2} = \left(\frac{n+1}{n+4}\right) I_n$, 4marks

(b) Hence evaluate $\int_0^1 x^4 \sqrt{1-x^2} dx$ in terms of I_0 . 2marks

(ii) Given that $f(x) = x^3$ and $g(x) = 3x-1$ show that the equation $f(x) + g(x) = 0$ has a unique solution in $\mathbb{R}$ that lies in the interval $[0,1]$ 3marks

8.

(a) Expand $(1+x+x^2)^{\frac{1}{2}}$ as a series in ascending powers of x , up to and including term in x^2 3marks

(i) Hence, or otherwise, show that $(1-x+x^2)^{\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 + \dots$ 2 marks

(ii) Find $\lim_{x \rightarrow 0} \frac{(1+x+x^2)^{\frac{1}{2}} - 1}{(1-x+x^2)^{\frac{1}{2}} - 1}$ 2mark

(b) By using the Squeeze theorem, or otherwise, to test the convergence of the series

$\sum_{n=1}^{\infty} \left(\frac{n}{(n+1)(n+2)} \right) \cos\left(\frac{1}{n}\right)$ 3marks

9.

Consider the numerical function f , defined on $]-\infty, +\infty[$ by $f(x) = \frac{2x^2 - 3x}{x^2 - 3x + 3}$ and (C_f) its representative curve.

(a) Evaluate $\lim_{x \rightarrow +\infty} f(x)$ and deduce the horizontal asymptote to (C_f) 2marks

(b) Study the variations of f 3marks

(c) Draw up the variation table of f 2marks

(d) Study the relative position of (C_f) and its horizontal asymptote 3marks

(e) Sketch (C_f) 2marks

The function g is the restriction of f on $[3, +\infty[$ defined as $g: [3, +\infty[\rightarrow J$
 $x \mapsto f(x)$

(f) Show that g is a bijection onto an interval J to be determined 2marks.

(g) Hence sketch $(C_{g^{-1}})$, the curve of the inverse function of g 2marks

10.

(i) Consider the sequence (u_n) defined by $u_0 = \frac{5}{2}, u_{n+1} = 3 - \frac{n(3-u_n)}{2(n+1)}, \forall n \in \mathbb{N}$.

(a) Prove by mathematical induction that $u_n \leq 3$, for all $n \geq 0$ 3 marks

(b) Show that (u_n) is monotonic increasing. 3 marks

(c) Show that (u_n) is convergent and deduce its limit. 2 marks

Another sequence (v_n) is defined as $v_n = n(3-u_n)$ for all $n \geq 1$.

(d) Show that (v_n) is a geometric progression. 2 marks

(e) Express v_n in terms of n 2 marks

(f) Hence deduce an expression of u_n in terms of n . 2 marks

(ii) Given the similarity transformation f over the set of complex numbers defined by

$$z' = (\alpha - i)z + \beta, \text{ where } \alpha, \beta \in \mathbb{C} \text{ has scale factor } \frac{\sqrt{2}}{2}, \text{ and angle } -\frac{\pi}{4}.$$

(a) Determine the complex number α .

2marks

(b) The centre of this similarity transformation is at the point $\Omega(1,2)$, determine the complex number β

3marks

END!



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