



**LITTORAL MATHEMATICS TEACHERS ASSOCIATION
(L.I.M.T.A)**



GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION

0775 FURTHER MATHEMATICS 2

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Subject Title	Further Mathematics
Paper No.	2
Subject Code No.	0775

THREE HOURS

Full marks may be obtained for answers to ALL questions.

Mathematical Formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. Given that the differential equation

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2e^x$$

(i) Find the complementary solution of the differential equation.

3 marks

(ii) Find also the particular integral of the differential equation.

3 marks

Hence, state the general solution of the differential equation.

1 mark

2. By using the definition

$$\sec h^{-1}(x) = \ln \left[\frac{1 + \sqrt{1 - x^2}}{x} \right] \text{ for all } x \in]0, 1]$$

(i) Show that $\sec h^{-1}(1 - x) = \ln \left[\frac{1 + \sqrt{2x - x^2}}{1 - x} \right]$ for all $x \in]0, 1]$

2 marks

Hence or otherwise, show that

(ii) $\frac{d}{dx} [\sec h^{-1}(1 - x)] = \frac{1}{(1 - x)\sqrt{2x - x^2}}$.

3 marks

(iii) $\int_0^{\frac{1}{2}} \frac{1}{(1 - x)\sqrt{2x - x^2}} dx = \ln[2 + \sqrt{3}]$

3 marks

3. Given that $I_n = \int_0^1 x^n (\sqrt{1 + x}) dx$, $n \in \mathbb{N}$,

(i) show that $(2n + 3)I_n = 4\sqrt{2} - 2nI_{n-1}$ for all $n \geq 1$.

4 marks

(ii) Hence evaluate $\int_0^1 x^2 (\sqrt{1 + x}) dx$

3 marks

4. Consider the curve with polar equation

$$r = \frac{2}{1 + \sin \theta}, \quad \theta \neq (-1)^{n+1} \left(\frac{\pi}{2} \right) + n\pi.$$

(i) Show that the Cartesian equation of this curve is $4y + x^2 - 4 = 0$.

Hence or otherwise,

(ii) Show that the polar equation of the tangent to this curve at the point with polar coordinates $\left(\frac{4}{3}, \frac{\pi}{6} \right)$ is

$$r = \frac{4}{\sqrt{3} \cos \theta + 3 \sin \theta}$$

4 marks

5.

(a) Describe the locus of the points $(x, y) \in \mathbb{R}^2$ such that the structure (S, X_c) where $S = \{1, x + iy, x - iy\}$ forms a group under multiplication over the set of complex numbers.

3 marks

- (b) Show that the arc length of the curve with parametric equation

$$x = 1 + t, \quad y = t^2 \quad \text{where } 0 \leq t \leq \frac{1}{2}$$

is given by $\frac{1}{4}[p + \ln(1 + p)]$ where p is a constant to be determined.

5 marks

6.

- (a) Given that S is a similitude transformation defined over the set of complex numbers by

$$z' = (1 + \sqrt{3}i)z + 3\sqrt{3} - 2\sqrt{3}i$$

- (i) Find the image of the complex number $z = 2 + 3i$

2 marks

- (ii) State the scale factor, centre and angle of the similitude transformation

4 marks

- (b) Given the vectors

$$\mathbf{p} = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} 7 \\ -2 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ t \end{pmatrix}$$

where t is a scalar parameter.

Determine the value(s) of t for which $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are linearly independent.

3 marks

7. Solve in $\mathbb{N}$, the system of equations

$$\begin{cases} 7x \equiv 3 \pmod{5} \\ 5x \equiv 3 \pmod{9} \end{cases}$$

6 marks

8.

- (a) If w^3 is a complex root of 1, find

(i) $(1 + 3w)(1 + 3w^2)$

2 marks

(ii) $(1 + 3w + w^2)$

2 marks

- (b) Given the complex number $z = \frac{1}{2}[\cos \theta + i \sin \theta]$ such that $z^n = \frac{1}{2^n}[\cos n\theta + i \sin n\theta]$.

By using Demoivre's theorem or otherwise, show that

$$\sum_{n=0}^{\infty} \frac{\cos n\theta}{2^n} = \frac{4 - 2 \cos \theta}{5 - 4 \cos \theta}$$

3 marks

9.

- (a) Given the function f defined by

$$f(x) = \begin{cases} \frac{2e^x}{e^x - 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

- (i) Study the differentiability of f at $x = 0$

2 marks

- (ii) Show that the point $(0, 1)$ is the centre of symmetry to the curve of f

2 marks

- (b) Consider the sequence (u_n) defined recursively by

$$\begin{cases} u_0 = 1 \\ u_n = 4 \\ 2u_{n+1} = 3u_n + 5u_{n-1} \end{cases}.$$

(i) Show that $2u_{n+1} - 5u_n = (-1)^n [2u_1 - 5u_0]$ **3 marks**

(ii) Show that (u_n) diverges **2 marks**

Show further that

(iii) $u_n \geq 1$ for all $n \in \mathbb{N}$ **2 marks**

(iv) (u_n) is monotonic increasing. **2 marks**

Two other sequences (v_n) and (w_n) are defined by $v_n = u_n + u_{n-1}$; $w_n = 2u_n - 5u_{n-1}$

(v) Show that (v_n) and (w_n) are geometric progressions **3 marks**

(vi) Express (v_n) and (w_n) in terms of n **2 marks**

10.

(a) A function f whose curve is denoted by C_f , is defined by

$$f(x) = \frac{x+1}{\sqrt{x^2+1}} - 1$$

By using the fact that $f(x) = \frac{x\left(1+\frac{1}{x}\right)}{|x|\sqrt{1+\frac{1}{x^2}}} - 1$

(i) Evaluate $\lim_{n \rightarrow +\infty} f(x)$ and $\lim_{n \rightarrow -\infty} f(x)$ **3 marks**

(ii) Deduce the equation of the asymptotes to C_f . **1 mark**

(iii) Study the variation of f **3 marks**

(iv) Study the relative position of C_f and the line $y = x$. **3 marks**

(v) Draw a variation table of C_f . **2 marks**

(vi) Sketch C_f , showing clearly the behavior of f as it approaches the asymptotes. **2 marks**

(b) The points A , B and C have position vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = k\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$ and $\mathbf{c} = \mathbf{j} - \mathbf{k}$ respectively, where k is a constant. Given that π is the Cartesian equation of the plane containing the points A , B and C , where $-2x - 3y + z = 4$.

(i) Find the value of k .

A line l has equation $\mathbf{r} = 5\mathbf{j} + 17\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 8\mathbf{k})$. **3 marks**

(ii) Find the coordinates of the point of intersection of the plane π and the line l . **3 marks**

END!