

GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

Pure Maths With Mechs 2
0765/2

JUNE 2022

ADVANCED LEVEL

Subject Title	Pure Mathematics With Mechanics
Paper No.	Paper 2
Subject Code No.	0765

Three hours.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

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7. Given that $f(x) = \frac{x^2+18}{x(x^2+9)}$,
express $f(x)$ in partial fractions.
Hence or otherwise, show that

$$\int_2^4 f(x) dx = \ln\left(\frac{4\sqrt{13}}{5}\right).$$

(3, 4 marks)

8. (i) Solve for x in the equation $4^x + 2^{2+x} = 12$.

(3 marks)

- (ii) The determinant of a matrix M is 45, where

$$M = \begin{pmatrix} \alpha & 3 & 5 \\ 4 & \alpha-1 & 6 \\ 1 & 4 & 0 \end{pmatrix}.$$

- (a) Show that $\alpha = 2$.

(2 marks)

- (b) Find M^{-1} , the inverse of the matrix M .

Hence, solve the system of equations

$$2x + 3y + 5z = 30$$

$$4x + y + 6z = 27$$

$$x + 4y = 45$$

(6 marks)

9. (a) Solve the differential equation $(x+2)^2 \frac{dy}{dx} + 3y^2 = 0$, given that $y = -2$ when $x = 0$.

(4 marks)

- (b) Sketch the curve $y = \frac{x+2}{x-1}$, showing clearly the points where the curve meets the coordinate axes and the behaviour of the curve as it approaches its asymptotes.

(5 marks)

10. (i) Given that p and q are two statements,
show that $p \Rightarrow q$ is logically equivalent to $\sim(p \wedge \sim q)$.

(3 marks)

- (ii) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = 2x^3 + 3x - 2$.

- (a) Show that f is monotonic increasing.

(2 marks)

- (b) Deduce that the equation $f(x) = 0$ has a root within the interval $0 < x < 1$.

(2 marks)

Taking $x = 0.5$ as first approximation to the root of the equation $f(x) = 0$, use Newton-Raphson's method once to obtain, to 2 decimal places, a second approximation to the root of this equation.

(3 marks)



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1. (i) Given that the roots of the equation $3x^2 - 4x + 2 = 0$ are α and β ,
 - (a) show that $\left(\alpha + \frac{\alpha}{\beta}\right)\left(\beta + \frac{\beta}{\alpha}\right) = 3$. (3 marks)
 - (b) find an equation, with integral coefficients, whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. (3 marks)
 - (ii) The polynomial $P(x)$ is such that $P(x) \equiv ax^3 + bx^2 - 2x - 6$. Given that $P(x)$ leaves a remainder of 10 when divided by $x - 2$ and that $2x - 3$ is a factor of $P(x)$, find the values of the constants a and b . (6 marks)
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2. (i) A function $f: \mathbb{R} - \{-1\} \rightarrow \mathbb{R} - \{1\}$, is defined by $f(x) = \frac{x-1}{x+1}$.
 - (a) Show that f is surjective. (3 marks)
 - (b) Find the composite function $(f \circ f)(x)$, stating its domain. (3 marks)
 - (c) Determine the parity of $(f \circ f)(x)$. (2 marks)
 - (ii) Expand $\frac{1-x}{(1+2x)^3}$ as a series in ascending power of x as far as term in x^2 . (4 marks)
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3. The position vectors of the points A, B and C are $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively, where $\mathbf{a} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = -3\mathbf{i} - \mathbf{j} + 7\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j}$. Given that M is the midpoint of AB , find
 - (a) an equation of the line AB , in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, (2 marks)
 - (b) the position vector of M , (2 marks)
 - (c) the Cartesian equation of the plane ABC , (5 marks)
 - (d) the value of the constant a for which the line $\mathbf{r} = \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + a\mathbf{j} + \mathbf{k})$ is parallel to the plane ABC . (2 marks)
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4. Given that $f(\theta) = \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta}$,
 - (a) prove that $f(\theta) \equiv \cos \theta + \sin \theta$. (3 marks)
 - (b) express $f(\theta)$ in the form $R \sin(\theta + \lambda)$, where $R > 0$ and λ an acute angle. (3 marks)
 - (c) find the general solutions of the equation $f(\theta) = 1$. (3 marks)
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5. (i) The sum of the sixth and the eighth term of an AP is 40. The seventh term is 4 times the second term. Find,
 - (a) the first term and the common difference, (3 marks)
 - (b) the least number of terms required for the sum of the progression to exceed 1000. (3 marks)
 - (ii) If
 - (c) $y = (\sin x)(\ln x)$, find $\frac{dy}{dx}$. (2 marks)
 - (d) $x = 1 + t^3$ and $y = t + t^4$, find $\frac{dy}{dx}$ in terms of t . (2 marks)
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6. (i) Given that z and w are two complex numbers, solve the simultaneous equations

$$3z + w = 9 + 11i$$

$$iw - z = -8 - 2i$$
, leaving the answers in the form $a + bi$. (4 marks)
 - (ii) Let $\mathbb{N}$ be the set of natural numbers. A relation R is defined on $\mathbb{N} \times \mathbb{N}$ by $(a, b)R(c, d) \Leftrightarrow \frac{d}{b} = \frac{c}{a}$. Show that R is an equivalence relation. (7 marks)