

LITTORAL MATHEMATICS TEACHERS ASSOCIATION

(L.M.T.A)



GENERAL CERTIFICATE OF EDUCATION REGIONAL MOCK EXAMINATION

0765 MATHS WITH MECHANICS 3

MARCH 2023

ADVANCED LEVEL

Subject Title	Pure Mathematics With Mechanics
Paper No.	3
Subject Code No.	0765

THREE HOURS

Full marks may be obtained for answers to ALL questions.

All questions carry equal marks.

You are reminded of the necessity for good English and orderly presentation in your answers.

Mathematical formulae Booklet published by the Board is allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

Calculators are allowed.

Start each question on a new page.

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1. The position vector $\mathbf{r}$ of a particle P of mass 1 kg at time t is given by

$$\mathbf{r} = [t^3 - 3t \mathbf{i} + t^2 + t \mathbf{j}] \text{ m}.$$

Find:

- The momentum of P at time $t = 3$. (4 marks)
- The kinetic energy of P at time $t = 3$. (2 marks)
- The work done by P in the interval $1 \leq t \leq 3$. (3 marks)
- The value of t , to one decimal place, for which the velocity of P is perpendicular to the vector $\mathbf{i} - \mathbf{j}$. (4 marks)

2. Two smooth spheres A and B each of mass m , and equal radii, moving towards each other with speeds $2u$ and u respectively collide. Given that e is the coefficient of restitution between the spheres and that after collision, the speed of sphere B is $\frac{3u}{2}$:

- Show that $e = \frac{2}{3}$. (4 marks)

Given that the impact lasts for 0.2 second, find in terms of m and u :

- The magnitude of the force exerted on B by A . (3 marks)
- Find the magnitude of the impulse exerted on sphere A due to the collision. (3 marks)
- Find also the loss in kinetic energy due to the impact. (3 marks)

3. A particle P is projected from a point O on the Cartesian plane with speed $3\sqrt{10} \text{ ms}^{-1}$. Given that P passes through a point A , which is at a horizontal distance of 3 meters from O , and 4 meters vertically above O , show that:

- $\tan \theta = 3$. (4 marks)
- The vertical displacement of P at time t is given by $h \ t = 9t - 5t^2$. (3 marks)

Hence or otherwise, find, to one decimal place:

- The maximum height attained by P . (2 marks)
- The time of flight of P . (2 marks)
- The horizontal distance covered by P . (2 marks)

(Take $g = 10 \text{ ms}^{-2}$)

4. i. Figure 2 below shows a particle A of mass m on a smooth horizontal plane connected by means of a light inextensible string, to another particle B of mass $2m$, which is on a rough plane inclined at an angle 60° to the horizontal. The coefficient of friction between B and the plane is 0.2. The system is released from rest with the string taut.

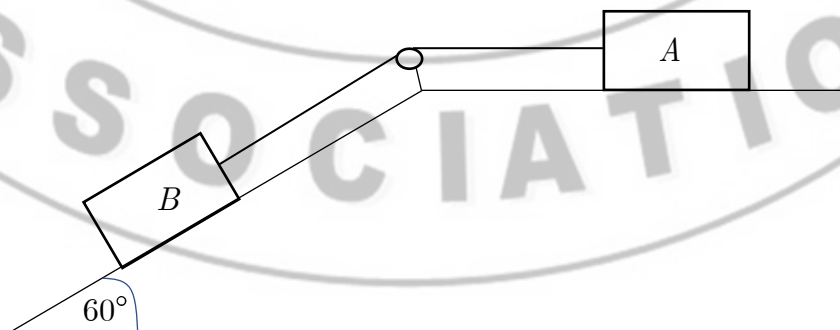


Figure 2

Find:

- The acceleration of the system. (4 marks)
- The tension in the string. (2 marks)
- The force exerted at the pulley. (2 marks)

(Take $g = 10 \text{ ms}^{-2}$)

- A particle starts from rest and moves such that its acceleration, a , is given by $a = \frac{1}{2} 4s + 1 \text{ ms}^{-2}$, where s is the displacement of the particle in meters. Find the velocity of the particle when it is 4 meters from its initial position. (5 marks)

- A uniform ladder AB of length $4a$ and weight W rests, with the end A against a smooth vertical wall and the end B on a rough horizontal ground. When a man of weight λ ascends a distance of $3a$ up the ladder, the ladder begins to slip. Given that the coefficient of friction between the ladder and the ground is $\frac{2}{5}$ and ladder makes an angle of θ with the ground, where $\tan \theta = \frac{13}{8}$:
 - Show that $\lambda = \frac{3}{2} W$. (4 marks)

Find:

 - The magnitude of the resultant force at the foot of the ladder. (2 marks)
 - To one decimal place, the angle of friction at the foot of the ladder. (2 marks)
 - An elastic string of natural length 3 meters and modulus of elasticity 60 newton obeys Hooke's Law. The string is stretched between two points which are 3.5 meters apart.
 - Find the tension in the string. (2 marks)
 - Find the energy stored in the string when it is stretch from a length of 3.7 meters to 3.9 meters. (3 marks)

- In the figure 1 below (not drawn to scale), the curve OP is a portion of the graph of $y^2 = x^3$ and PQR is a right-angle triangle with right angle at R .

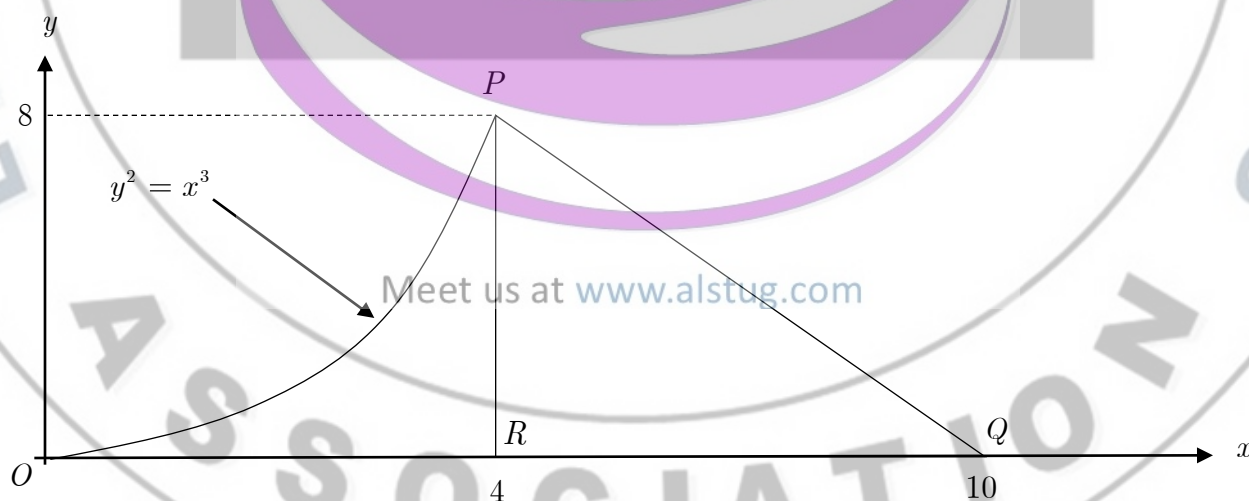


Figure 1

- Show by integration that the Cartesian coordinates of the centroid of portion OPR are $\left(\frac{20}{7}, \frac{5}{2}\right)$ from Oy and Ox respectively. (6 marks)

- ii. Determine the coordinates of the centre of gravity of the portion $OPQR$ from the axes Oy and Ox . (5 marks)
- iii. The lamina is freely suspended from the end Q . Show that when OQ makes an angle of θ with the vertical, then $\tan \theta = \frac{21}{41}$. (2 marks)

- 7.
- i. A car of mass 1200 kg tows a carriage of mass 800 kg by means of a tow rope, when moving on a straight horizontal road. The resistance to the motion of the car and the carriage are 500 N and 300 N respectively. Given that engine of the car works at a constant rate of 45 kW and that the car moves at a speed of 25 m/s, find:
- The acceleration of the car and carriage. (4 marks)
 - The tension in the tow rope. (2 marks)
 - The maximum speed of the car on the level road. (3 marks)
- ii. A particle moves in a horizontal circle of radius 0.5 m, turning at a rate of 240π rads per minute. Find the radial acceleration of the particle. (4 marks)

- 8.
- i. The events A and B are such that $P(A) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{3}$ and $P(A|B) = \frac{4}{9}$. Find:
- $P(B)$ (2 marks)
 - $P(A \cup B)$ (2 marks)
 - $P(B|A')$ (2 marks)
- ii. When Jane is bored, she can be entertained by phone, by TV or by computer, with probabilities $\frac{3}{5}$, $\frac{2}{15}$ and $\frac{4}{15}$ respectively. The probabilities that she finds something interesting on phone, TV or computer are $\frac{7}{10}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. Find the probability that:
- She finds something interesting. (3 marks)
 - She is entertained by her phone or she finds something interesting. (2 marks)
 - She is entertained by her phone given that she finds something interesting. (2 marks)

END

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