

WEST MATHEMATICS TEACHERS' PEDAGOGIC GROUP

GENERAL CERTIFICATE OF EDUCATION MOCK EXAMINATION

0775 Further Mathematics 1

23rd MARCH, 2023

ADVANCED LEVEL

Centre Number	
Centre Name	
Candidate Identification No.	
Candidate Name	

Mobile phones are NOT allowed in the examination room.

MULTIPLE CHOICE QUESTION PAPER

One and a half hours

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before you start answering the questions in this paper. Make sure you have a soft HB pencil and an eraser for this examination.

1. USE A SOFT HB PENCIL THROUGHOUT THE EXAMINATION.
2. DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

Before the examination begins:

3. Check that this question booklet is headed **Advanced Level 0775 Further Mathematics 1**.

4. Fill in the information required in the spaces above.

5. Fill in the information required in the spaces provided on the answer sheet using your HB pencil:

Candidate Name, Exam Session, Subject Code and Candidate Identification Number.

Take care that you do not erase or fold the answer sheet or make any marks on it other than those asked for in these instructions. *How to answer the questions in this examination:*

6. Answer **ALL** the 50 questions in this Examination. All questions carry equal marks.

7. Calculators are allowed.

8. Each question has FOUR suggested answers: **A, B, C** and **D**. Decide which answer is appropriate.

Find the number of the question on the Answer Sheet and draw a horizontal line across the letter to join the square brackets for the answer you have chosen.

For example, if **C** is your correct answer, mark **C** as shown below:

[A] [B] **[C]** [D]

9. Mark only one answer for each question. If you mark more than one answer, you will score a zero for that question. If you change your mind about an answer, erase the first mark carefully, and then mark your new answer.
10. Avoid spending too much time on any one question. If you find a question difficult, move on to the next question. You can come back to this question later.
11. Do all rough work in this booklet using the blank spaces in the question booklet.
12. **At the end of the examination, the invigilator shall collect the answer sheet first then the question booklet. DO NOT ATTEMPT TO LEAVE THE EXAMINATION HALL WITH IT.**

MARCH 2023 / MTPG / 0775 / 1 / C

1. If $y = \lambda x e^{-x}$ satisfies the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 3e^{-x}$, then the value of λ is

- A $\frac{1}{2}$
- B $-\frac{1}{2}$
- C 2
- D -2

2. $\frac{4x^2 + 3}{(1-x)^2(1-x^2)}$ can be expressed in partial fractions where A, B, C and D are real constants, as

- A $\frac{A}{x-1} + \frac{B}{(1-x)^2} + \frac{Cx+D}{1-x^2}$
- B $\frac{Ax+B}{(1-x)^2} + \frac{Cx+D}{1-x^2}$
- C $\frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1+x} + \frac{D}{1-x}$
- D $\frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{(1-x)^3} + \frac{D}{1+x}$

3. If $z = 1 + \cos \alpha + i \sin \alpha$, then the argument of z^{10} is

- A 10α
- B α
- C 5α
- D 20α

4. The similarity transformation

$$z' = \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)z + (1 - i\sqrt{3})$$
 represents

- A A clockwise rotation of $\frac{\pi}{3}$ about $z_0 = 2$
- B An anticlockwise rotation of $\frac{\pi}{3}$ about $z_0 = 2$
- C A clockwise rotation of $\frac{\pi}{6}$ about $z_0 = 2$
- D An anticlockwise rotation of $\frac{\pi}{6}$ about $z_0 = 2$

5. If $|z + 2 + 3i| = 4$, then the least value of $|z + 7 - 9i|$ is

- A 4
- B 9
- C 13
- D 17

6. Which of the following sets of vectors forms a basis for $\mathbb{R}^2$?

- A $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\}$
- B $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\}$
- C $\left\{ \begin{pmatrix} -3 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \end{pmatrix} \right\}$
- D $\left\{ \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \end{pmatrix} \right\}$

7. Given that the series $\sum_{r=1}^{\infty} \left(\frac{1}{r}\right)^{3k}$ converges, then

- A $k \leq 3$
- B $k > 3$
- C $k \leq \frac{1}{3}$
- D $k > \frac{1}{3}$

8. The expansion of $\ln \left(\frac{1+2x}{1-x}\right)$ is valid for x in the range

- A $-1 < x < 1$
- B $-1 \leq x < 1$
- C $-\frac{1}{2} \leq x \leq \frac{1}{2}$
- D $-\frac{1}{2} < x \leq \frac{1}{2}$

9. A sequence is defined recursively as $u_1 = 2$ and $u_{n+1} = \frac{1}{2}u_n$, $n \geq 1$, $u_n =$
- A 2^{n-1}
 B 2^{-n+1}
 C 2^{n-2}
 D 2^{-n+2}
14. The focus of the parabola $(y - 1)^2 = 2x + 6$ is
- A $\left(-\frac{7}{3}, 1\right)$
 B $\left(\frac{7}{3}, 1\right)$
 C $\left(\frac{5}{3}, 1\right)$
 D $\left(-\frac{5}{3}, 1\right)$

10. The first two terms in the Maclaurin series expansion of $e^x \cos x$ is

- A $x + \frac{1}{2}x^2$
 B $1 + x$
 C $x + \frac{1}{3}x^2$
 D $1 + \frac{1}{2}x^2$

15. If $f_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ and $f_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ are the permutations of the elements $\{1, 2, 3, 4\}$, then $f_1 \circ f_2 =$

- A $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$
 B $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$
 C $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$
 D $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

11. The general solution of the differential equation $x \sec^2 y \frac{dy}{dx} + \tan y = \cos x$ is

- A $x \tan y = -\sin x + k$
 B $x \tan y = \sin x + k$
 C $x \sec^2 y = -\sin x + k$
 D $x \sec^2 y = \sin x + k$

12. The polar equation $r = -8 \cos \theta$ represents

- A a parabola
 B an ellipse
 C a hyperbola
 D a circle

16. A subgroup of a group having 15 elements may have

- A 3 elements
 B 4 elements
 C 6 elements
 D 10 elements

13. The probability distribution for a discrete random variable X is

x	1	2	3
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{12}$

The mean value of X is

- A $\frac{1}{3}$
 B 1
 C 3
 D $\frac{13}{6}$

17. A continuous random variable X has probability density function f , where

$$f(x) = \begin{cases} \mu(1-x^2), & 0 \leq x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

The cumulative probability function $F(x) =$

- A $\begin{cases} 0, & x < 0 \\ \mu(1-x^2), & 0 < x \leq 1 \\ 1, & x > 1 \end{cases}$
- B $\begin{cases} \mu\left(x - \frac{x^3}{3}\right), & 0 < x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$
- C $\begin{cases} 0, & x < 0 \\ \mu\left(x - \frac{x^3}{3}\right), & 0 < x \leq 1 \\ 1, & x > 1 \end{cases}$
- D $\begin{cases} 0, & x < 0 \\ \mu\left(x - \frac{x^3}{3}\right), & 0 < x \leq 1 \\ 0, & x > 1 \end{cases}$

18. A particle of mass m kg is projected vertically upwards in a medium in which resistance to motion is kv per unit mass, where v is the velocity and k is a constant. The differential equation describing its motion is

- A $g - kv = \frac{dv}{dt}$
- B $-g - kv = \frac{dv}{dt}$
- C $g + kv = \frac{dv}{dt}$
- D $kv - g = \frac{dv}{dt}$

19. A particle moves in a straight line with simple harmonic motion of period π seconds. Given that the maximum velocity of the particle is 5 ms^{-1} , the amplitude is

- A $\frac{5}{4} \text{ m}$
- B $\frac{5}{2} \text{ m}$
- C 5 m
- D 10 m

20. If a particle moves along a straight line Ox such that its displacement x metres from O at time t seconds is given by $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 4x = 0$, then its period of motion is

- A 2π
- B π
- C $\frac{4\pi}{\sqrt{7}}$
- D $\frac{\pi}{\sqrt{7}}$

21. The domain of definition of the function $f(x) = \frac{\ln(1+x)}{1-x}$ is

- A $] -1, 1[$
- B $] -1, 1[\cup] 1, +\infty[$
- C $] -\infty, -1[\cup] 1, +\infty[$
- D $] -\infty, 1[\cup] 1, +\infty[$

22. A function f is such that

$$e^{-x} + \frac{2x}{x+2} \leq f(x) \leq \frac{2x^2-3}{1+x^2}, \quad \lim_{x \rightarrow +\infty} f(x) =$$

- A 2
- B -3
- C $-\infty$
- D $+\infty$

23. Which of the following is TRUE about the function $h(x) = \frac{(x+5)(x-3)(2x+1)}{(x+1)(x-3)}$?

- A $\lim_{x \rightarrow 3} h(x) = 0$
- B $x = 3$ is a vertical asymptote of $y = h(x)$
- C h is continuous at $x = 3$
- D h has a removable discontinuity at $x = 3$

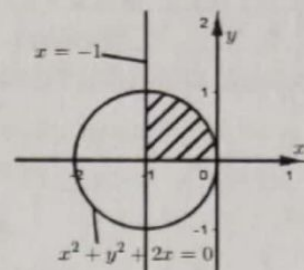
24. If $f(x) = x + [x]$, where $[x]$ is the greatest integer $\leq x$, then $\int_{-1}^1 f(x) dx =$

- A 1
- B 2
- C 3
- D $\frac{3}{2}$

25. $\lim_{x \rightarrow 1} \arccos \left(\frac{1 - \sqrt{x}}{1 - x} \right) =$

- A $\frac{\pi}{3}$
- B $\frac{\pi}{6}$
- C $\frac{\pi}{2}$
- D 0

29. The inequalities representing the shaded region below are



26. A force $\mathbf{F}$ of magnitude 7 N acting in the direction $(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ moves a particle from a point A to a point B whose position vectors are $(\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$ m and $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m respectively. The work done by this force is

- A 12 J
- B 24 J
- C 4 J
- D 25 J

- A $[(x, y) \in \mathbb{R}^2 : x \leq 1, y \geq 0, x^2 + y^2 + 2x \geq 0]$
- B $[(x, y) \in \mathbb{R}^2 : x \geq -1, y \geq 0, x^2 + y^2 + 2x \leq 0]$
- C $[(x, y) \in \mathbb{R}^2 : x \geq 1, y \geq 0, x^2 + y^2 + 2x \geq 0]$
- D $[(x, y) \in \mathbb{R}^2 : x \leq -1, y \leq 0, x^2 + y^2 + 2x \geq 0]$

27. The force $\mathbf{F}$ acts along the line whose vector equation is $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. The vector moment of $\mathbf{F}$ about the origin O is

- A $\mathbf{a} \cdot \mathbf{F}$
- B $\mathbf{a} \times \mathbf{F}$
- C $\mathbf{b} \times \mathbf{F}$
- D $\mathbf{b} \cdot \mathbf{F}$

30. Given the statements P : John had 25 points and q : He wrote 5 papers. Then $\sim(p \implies q)$ in ordinary English is

- A If John had 25 points, then he did not write 5 papers
- B If John did not write 5 papers, then he did not have 25 points
- C John had 25 points and he did not write 5 papers
- D John did not have 25 points and he did not write 5 papers

31. The contrapositive of the statement: "if $x + 2 = 7$, then $x = 5$ " is

- A If $x + 2 \neq 7$, then $x \neq 5$
- B If $x \neq 5$, then $x + 2 \neq 7$
- C If $x = 5$, then $x + 2 = 7$
- D If $x \neq 5$, then $x + 2 = 7$

28. The forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act at the points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively.

Given that $\sum_{i=1}^3 \mathbf{F}_i \neq \mathbf{0}$ and $\sum_{i=1}^3 \mathbf{r}_i \times \mathbf{F}_i = \mathbf{0}$, then this system of forces

- A is in equilibrium
- B reduces to a couple
- C reduces to a single force through O
- D reduces to a single force and a couple

32. Given that the velocities of a sphere of mass $3m$ before and after impact with another sphere are $7\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ and $-\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ respectively, then the line joining the centres of these spheres at impact is parallel to the vector

- A $\mathbf{i}$
- B $\mathbf{j} + \mathbf{k}$
- C $7\mathbf{i} + \mathbf{j} - 4\mathbf{k}$
- D $8\mathbf{i} - 5\mathbf{j} - 6\mathbf{k}$

33. Given that $\sinh x = \frac{5}{12}$, then the value of $\cosh x$ is

- A $+\frac{13}{12}$
- B $-\frac{13}{12}$
- C $\pm\frac{13}{12}$
- D $\frac{169}{144}$

34. Which of the functions below is NOT injective?

- A $\sinh x, x \in \mathbb{R}$
- B $\cosh x, x \in \mathbb{R}$
- C $\tanh x, x \in \mathbb{R}$
- D $\operatorname{artanh} x, x \in]-1, 1[$

35. The matrix $M = \begin{pmatrix} -1 & 2 & 1 \\ 2 & 4 & -2 \\ 1 & -2 & -1 \end{pmatrix}$ maps the 3-dimensional space to

- A a plane
- B a line
- C the origin
- D a point

36. Given that the vector $\mathbf{a}$ lies in the plane of vectors $\mathbf{b}$ and $\mathbf{c}$, which of the following is correct?

- A $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 1$
- B $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$
- C $\mathbf{a} \cdot \mathbf{b} = 0, \mathbf{a} \cdot \mathbf{c} = 0$
- D $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$

37. The vertices A, B and C of $\triangle ABC$ have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. The area of $\triangle ABC$ is

- A $\frac{1}{2}|\mathbf{a} \times \mathbf{b}|$
- B $\frac{1}{2}|\mathbf{a} \times \mathbf{c}|$
- C $\frac{1}{2}|\mathbf{a} \times \mathbf{b} \times \mathbf{c}|$
- D $\frac{1}{2}|(\mathbf{a} - \mathbf{b}) \times (\mathbf{b} - \mathbf{c})|$

38. The table below shows the velocity of a cyclist at given intervals of time

times (s)	0	1	2
velocity (m/s)	0	5	7

Using Simpson's rule, the distance covered after 2 seconds is

- A 9 m
- B $\frac{17}{3}$ m
- C $\frac{17}{2}$ m
- D $\frac{27}{2}$ m

39. Using the approximation $y_{n+1} \approx y_n + h \left(\frac{dy}{dx} \right)_n$ and a step length of 0.2, the approximate value of y when $x = 1.4$ given that $\frac{dy}{dx} = 2x - y$ and $y = 1$ when $x = 1$ is

- A 1.44
- B 0.44
- C 1
- D 1.24

40. The equation of the tangent at the pole to the curve with equation $r = a(1 - 2\sin \theta)$ is given by $\theta =$

- A $\frac{\pi}{3}$
- B $\frac{\pi}{2}$
- C $\frac{\pi}{6}$
- D 0

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41. A uniform rod of length $2a$ and mass m oscillates about its position of stable equilibrium with equation of motion $2a\left(\frac{d\theta}{dt}\right)^2 = 3g \cos \theta$, where θ is the angle made by the rod with the downward vertical. The period of small oscillations of the rod is

- A $\pi\sqrt{\frac{a}{3g}}$
 B $2\pi\sqrt{\frac{a}{3g}}$
 C $4\pi\sqrt{\frac{a}{3g}}$
 D $2\pi\sqrt{\frac{2a}{3g}}$

42. A flywheel whose moment of inertia is 50 kgm^2 is rotating at 4 rads^{-1} . The kinetic energy of the wheel is

- A 200 J
 B 400 J
 C 800 J
 D 1600 J

43. The remainder when -95 is divided by 6 is

- A 4
 B -1
 C 1
 D -4

44. Which of the following congruences has no solution?

- A $8x \equiv 6 \pmod{12}$
 B $6x \equiv 5 \pmod{7}$
 C $21x \equiv 15 \pmod{33}$
 D $18x \equiv 12 \pmod{20}$

45. A particle P moves round a curve with polar equation $r = ke^{2\theta}$ with constant angular velocity ω . The radial component of the acceleration of P is

- A $\omega^2 r$
 B $2\omega^2 r$
 C $3\omega^2 r$
 D $4\omega^2 r$

46. $\int_0^\pi |\cos x| e^{\sin x} dx =$

- A $2(e-1)$
 B $e^2 - 1$
 C 0
 D 2

47. The arc length of the curve $f(x) = \cosh x$, for $0 \leq x \leq \ln 2$ is

- A $\frac{3}{4}$
 B $\frac{5}{4}$
 C $\frac{5}{2}$
 D $\frac{3}{2}$

48. $\int \frac{1}{\sqrt{9-4x^2}} dx =$

- A $\frac{1}{2} \arccos\left(\frac{3x}{2}\right) + K$
 B $\frac{1}{2} \arcsin\left(\frac{2x}{3}\right) + K$
 C $\frac{2}{3} \arccos\left(\frac{2x}{3}\right) + K$
 D $\frac{4}{9} \arccos\left(\frac{2x}{3}\right) + K$

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$\frac{1}{2} \arcsin\left(\frac{2x}{3}\right) + K$

49. If for any integer n , $\int_n^{n+1} f(x) dx = n^2$, then

$$\int_2^4 f(x) dx =$$

- A 16
- B 18
- C 21
- D 13

50. The rate of decay of a substance is cx , where x is the amount of substance remaining at time t and c is a constant. The time, T , in which the amount of the substance is halved is

- A $T = c \ln 2$
- B $T = \frac{1}{c} \ln 2$
- C $T = \ln 2c$
- D $T = \frac{1}{2} \ln 2$

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